A generalized mapping procedure of ductile fracture model between stress and strain spaces

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Abstract. A frequently applied mapping method for a ductile fracture model from stress space to strain space is based on the isotropic von Mises model which is not always consistent with non-quadratic yield criterion used for modelling of ductile fracture. The proposed research work demonstrates a general procedure which can be incorporated with any yield criterion in ductile fracture mechanics. A new mapping method of fracture model from stress space to strain space has also been formulated in this study. As an application, a modified Mohr Coulomb fracture model has been proposed by combining Hosford with the mean stress.

1 A modified Mohr-Coulomb fracture model

The Mohr-Coulomb fracture model assumes that the fracture takes place when a critical value is reached by the combination of shear stress and normal stress:

$$\sigma_{MC} = \tau + \mu \sigma_n$$

(1)

where $\sigma_{MC}$ and $\mu$ are two material parameters; $\tau$ is the shear stress; $\sigma_n$ is the normal stress which is defined by principal stresses:

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2}$$

(2)

where $\sigma_1$ and $\sigma_3$ are maximum and minimum principal stresses.

In this study, a modified Mohr-Coulomb fracture model with the Hosford and mean stress will be applied. It is formulated by the following equation:

$$\sigma_{MC} = \bar{\sigma}_h + \mu \sigma_m$$

(3)

where $\bar{\sigma}_h$ is the Hosford stress:

$$\bar{\sigma}_h = \left(\frac{|\sigma_1 - \sigma_2| + |\sigma_1 - \sigma_3| + |\sigma_2 - \sigma_3|}{2}\right)^{1/m}$$

(4)

$\sigma_m$ is the mean stress which is defined by principal stresses:

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

(5)

2 Transformation from space $(\sigma_1, \sigma_2, \sigma_3)$ to $(\eta, L, \bar{\sigma})$

The proposed fracture model is a stress based criterion. Fracture loci predicted in principal stress space $(\sigma_1, \sigma_2, \sigma_3)$ can be transformed to principal strain space $(\epsilon_1, \epsilon_2, \epsilon_3)$, which consists of two transformation steps: 1) from $(\sigma_1, \sigma_2, \sigma_3)$ to $(\eta, L, \bar{\sigma})$; 2) from $(\eta, L, \bar{\sigma})$ to $(\epsilon_1, \epsilon_2, \epsilon_3)$.

The stress triaxiality $\eta$ and the Lode parameter $L$ are defined as follows:

$$\eta = \frac{\sigma_m}{\bar{\sigma}}$$

(6)

$$L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

(7)

where $\bar{\sigma}$ is the equivalent stress defined by the material model. According to Bai [1] and Lou [2], in the first transformation step, the principal stresses were formulated in terms of $\eta$, $L$, $\bar{\sigma}$ with the Von Mises criterion, which is defined as follow:

$$\bar{\sigma} = \left(\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}{2}\right)^{1/2}$$

(8)

However, for the many materials, the numerical simulation is performed with various yield criterion. The mapping procedure with isotropic Von Mises model may cause inconsistency with the stress updating. The following describes a generalized mapping procedure between principal stress and strain space with any yield function for the proposed fracture model.

Hosford [3] proposed a generalized isotropic yield criterion which is related to crystallographic theory. It is formulated as following:

$$|\sigma_1 - \sigma_2|^a + |\sigma_1 - \sigma_3|^a + |\sigma_2 - \sigma_3|^a = 2\bar{\sigma}^a$$

(9)

where $a$ is a crystallographic related parameter, which is equal to 6 for bcc materials, 8 for fcc materials.

With the definition of Lode parameter $L$ (Equation 7) and mean stress (Equation 5), $\sigma_2$ can be eliminated:
\[
\frac{\sigma_1 + \sigma_3}{2} = \bar{\sigma} - \frac{L(\sigma_1 - \sigma_3)}{6}
\]  \quad (10)

Then the Hosford formula can be written by:

\[
2\bar{\sigma}^a = |\sigma_1 - \sigma_3|^a \left[ \left(\frac{1-L}{L} \right)^a + \left(\frac{1+L}{L} \right)^a + 1 \right] + 1
\]  \quad (11)

Combine the Erreur! Source du renvoi introuvable, and Erreur! Source du renvoi introuvable, the principal stresses are formulated in space \((\bar{\eta}, L, \bar{\sigma})\) with Hosford coefficients:

\[
\sigma_1 = \bar{\sigma} - \frac{La\bar{\sigma}}{6} + \frac{a\bar{\sigma}}{2}
\]
\[
\sigma_2 = \bar{\sigma} + \frac{La\bar{\sigma}}{3}
\]
\[
\sigma_3 = \bar{\sigma} - \frac{La\bar{\sigma}}{3} - \frac{a\bar{\sigma}}{2}
\]
where \(a = \left( \frac{2}{\left( \frac{1-L}{L} \right)^a + \left( \frac{1+L}{L} \right)^a + 1} \right)^{1/a} \)  \quad (12)

Adding together the principal stresses Equation 12 into the proposed model Erreur! Source du renvoi introuvable, the fracture model can be written in space \((\bar{\eta}, L, \sigma)\):

\[
\sigma_{MC} = \bar{\sigma}[\bar{\sigma}_h + \mu\bar{\eta}]
\]  \quad (13)

where \(\bar{\sigma}_h = \left( \frac{|\sigma_1 - \sigma_2|^m + |\sigma_1 - \sigma_3|^m + |\sigma_2 - \sigma_3|^m}{2} \right)^{1/m} \)

\[
\left( \frac{a}{2} \frac{La}{2} \right)^m + \frac{\alpha}{m} \frac{\alpha}{m} + \frac{La}{2} \right)^{1/m}
\]

3 Transformation from \((\bar{\eta}, L, \bar{\sigma}^f)\) to \((\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3)\)

To map the fracture surface from \((\bar{\eta}, L, \bar{\sigma}^f)\) to \((\bar{\eta}, L, \bar{\varepsilon}^f)\), the equivalent plastic strain to fracture \(\bar{\varepsilon}^f\) at a given stress state of \((\bar{\eta}, L)\) is calculated from the equivalent stress to fracture \(\bar{\sigma}^f\) with a true stress-strain curve. A two parameters power hardening law is used in this paper.

\[
\bar{\sigma}^f = K(\bar{\varepsilon}^f)^n
\]  \quad (14)

where \(K\) and \(n\) are strain hardening coefficients which are usually determined by the uniaxial tensile test.

Equation 13 can be written in space \((\bar{\eta}, L, \bar{\varepsilon}^f)\):

\[
\bar{\varepsilon}^f = \left( \frac{\sigma_{MC}}{\bar{\sigma}_h + \mu\bar{\eta}} \right)^{-1/n}
\]  \quad (15)

Under proportional loading, the plastic strain tensor components to the fracture are proportional to the gradients of the plastic potential: