CYCLIC PLASTICITY MODEL WITH ANISOTROPY EVOLUTION

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Abstract. A framework for a combined anisotropic-kinematic hardening model of large-strain cyclic plasticity is presented, and the details of modeling which is based on the Yoshida–Uemori model are described. To describe the anisotropy evolution, the shape of the yield surface and that of the bounding surface are assumed in the model to change simultaneously. The model was validated by comparing the calculated results of stress–strain responses with experimental data of an aluminum sheet and cyclic straining on an advanced high-strength steel sheet.

1 Introduction

For accurate numerical simulation of sheet metal forming, the use of material models that properly describe plasticity behaviors, in terms of sheet anisotropy and the Bauschinger effect, is of vital importance.

Conventional plasticity models assume that $r$-values and flow stress directionality remain constant throughout the deformation. However, some metallic sheets exhibit significant changes in planar $r$-value anisotropy and flow stress directionality (e.g.,[1, 2]) and also the shape of yield surface (e.g.,[3, 4]) as plastic strain increases.

Another important issue in material modeling is describing the Bauschinger effect, especially for accurate numerical simulation of springback of high-strength steel (HSS) sheets (e.g.,[7-9]). Although many types of cyclic plasticity models have been proposed to describe the Bauschinger effect and cyclic hardening behavior, very few models are able to describe the anisotropy evolution (see articles [10, 11]).

The present authors recently published a paper on anisotropic hardening of sheet metals [11]. This paper describes a framework for a combined anisotropic-kinematic hardening modeling based on the Yoshida-Uemori model [5-7]. The model was validated by comparing the numerical prediction of sheet-direction-dependent hardening behavior to experimental results for an aluminum sheet and a 780-MPa advanced HSS (AHSS) sheet.

2 Anisotropic-kinematic hardening model

2.1 Modeling of anisotropy evolution

In the present paper, a model is constructed within the framework of the associated flow rule of plasticity. With the assumption of small elastic and large plastic deformation, the rate of deformation $\dot{D}$ is decomposed into its elastic and plastic parts, $\dot{D}^e$ and $\dot{D}^p$, respectively, as follows:

$$D = D^e + D^p. \quad (1)$$

The constitutive equation of elasticity is expressed as follows:

$$\sigma = C : (D - D^e) \quad (2)$$

where $\sigma$ and $\sigma^e$ are the Cauchy stress and its objective rate, respectively, and $C$ is the elasticity modulus tensor.

The initial yield criterion is expressed by the following equation:

$$f = \phi^0(\sigma) - Y = \sigma(\sigma) - Y = 0, \quad (3)$$

where $Y$ is the initial yield stress and $\sigma$ is the effective stress.

In kinematic hardening modeling, the yield function $f$ is given by the following equation:

$$f = \phi(\sigma - \alpha, \varepsilon) - Y = \sigma(\bar{\sigma}(\sigma, \varepsilon)) - Y = 0, \quad (4)$$

where $\alpha$ denotes the backstress. Based on the following definitions of the effective plastic strain and its rate

$$\bar{\sigma} = \sigma - \alpha, \quad \dot{\varepsilon} = \int_0^{\bar{\sigma}} dt, \quad (5)$$

the associated flow rule is written as follows:

$$D^p = \frac{\partial f}{\partial \sigma} \dot{\alpha} = \frac{\partial \phi}{\partial \sigma} \dot{\alpha}, \quad (6)$$

where $\dot{\alpha}$ ($\geq 0$) is the plastic multiplier. For a homogeneous yield function of degree one, $\phi$, the work conjugate type definition of the effective plastic strain...
rate (see Eq.(5)) reduces to \( \dot{\lambda} = \dot{\varepsilon} \). Thus, the elasto-plasticity constitutive equation can be written as follows:

\[
\sigma = C^{\text{ep}} : D, \quad \text{(7)}
\]

\[
C^{\text{ep}} = \begin{cases} 
C & \text{if } \lambda = 0 \\
C - \frac{C}{\partial \sigma} : \frac{C}{\partial \sigma} & \text{if } \lambda > 0
\end{cases}
\quad \text{(8)}
\]

where \( \dot{H} \) is the rate of workhardening.

In general, the evolution of anisotropy is expressed by the evolutionary change of the yield function \( \phi \) as a function of the effective plastic strain \( \varepsilon_{\text{ep}} \). In the present paper, the anisotropic hardening of the yield surface is expressed as follows:

\[
\phi(\sigma, \varepsilon) = \mu(\varepsilon) \phi_1(\sigma) + (1 - \mu(\varepsilon)) \phi_2(\sigma) \quad \text{for } \varepsilon_A \leq \varepsilon \leq \varepsilon_B
\quad \text{(9)}
\]

Here, \( \phi_1(\sigma) \) and \( \phi_2(\sigma) \) are two different yield functions defined at the effective plastic strains \( \varepsilon_A \) and \( \varepsilon_B \), respectively, and \( \mu(\varepsilon) \) is an interpolation function of the effective plastic strain, where

\[
1 = \mu(\varepsilon_A) \geq \mu(\varepsilon) \geq \mu(\varepsilon_B) = 0.
\quad \text{(10)}
\]

Note that the types of these two yield functions, \( \phi_1(\sigma) \) and \( \phi_2(\sigma) \), do not need to be the same. An advantage of this modeling framework is that, if the two yield functions \( \phi_1(\sigma) \) and \( \phi_2(\sigma) \) are convex, \( \phi(\sigma, \varepsilon) \) always satisfies the convexity. The derivatives are expressed as follows:

\[
\frac{\partial \phi}{\partial \sigma} = \mu(\varepsilon) \frac{\partial \phi_1}{\partial \sigma} + (1 - \mu(\varepsilon)) \frac{\partial \phi_2}{\partial \sigma},
\quad \text{(11)}
\]

\[
\frac{\partial \phi}{\partial \varepsilon} = (\phi_1(\sigma) - \phi_2(\sigma)) \frac{\partial \mu(\varepsilon)}{\partial \varepsilon}.
\]

If we have \( M \) sets of experimental data \((\sigma_0, \sigma_{45}, \sigma_{90}, \sigma_0, \sigma_{90}, \sigma_{0}, \sigma_{45}, \sigma_{90}, \sigma_{0}, \sigma_{45}, \sigma_{90}, \sigma_{0}, \sigma_{45}, \sigma_{90}, \sigma_{0}, \sigma_{45}, \sigma_{90})\), etc.) for material parameter identification corresponding to \( M \) discrete plastic strain points, \( \varepsilon_A, \varepsilon_{A+1}, \ldots, \varepsilon_i, \varepsilon_{i+1}, \ldots, \varepsilon_M \), we can determine \( M \) sets of yield functions \( \phi_i(\sigma) \), \( \phi_i(\sigma) \), \ldots, \( \phi_i(\sigma) \), \ldots. \( \phi_i(\sigma) \). Using an interpolation function \( \mu(\varepsilon) \), the yield function \( \phi(\sigma, \varepsilon) \) can be defined by the following equation:

\[
\phi(\sigma, \varepsilon) = \mu(\varepsilon) \phi_1(\sigma) + (1 - \mu(\varepsilon)) \phi_{i+1}(\sigma) \quad \text{(12)}
\]

for \( \varepsilon_i \leq \varepsilon \leq \varepsilon_{i+1} \).

The following nonlinear equation is proposed for use as the interpolation function:

\[
\mu(\varepsilon) = 1 - \left( \frac{\varepsilon - \varepsilon_i}{\varepsilon_{i+1} - \varepsilon_i} \right)^{p_i} \quad \text{for } \varepsilon_i \leq \varepsilon \leq \varepsilon_{i+1} \quad \text{(13)}
\]

where \( p_i \) (i = 1, 2, ..., \( M - 1 \)) are material constants.

Among the various types of anisotropic yield functions available, stress polynomial-type models (e.g., [12-15]) are suitable for use in modeling anisotropy evolution. A polynomial-type yield criterion is given by the following equation:

\[
f = \phi^{(m)}(\sigma) - \sigma_{y}^m = \sigma_{y}^m = 0. \quad \text{(14)}
\]

where \( \phi^{(m)}(\sigma) \) denotes the \( m \)-th order stress polynomial-type yield function. For example, when \( m = 6 \) (Yoshida et al. [15]) under plane stress condition,

\[
\phi^{(6)} = C_1 \sigma_1^6 - 3C_2 \sigma_1^4 \sigma_2^2 + 6C_3 \sigma_1^2 \sigma_2^4 - 7C_4 \sigma_2^6 + 6C_5 \sigma_1^6 - 4C_6 \sigma_1^4 \sigma_2^2 + 4C_7 \sigma_1^2 \sigma_2^4 - 2C_8 \sigma_2^6 + C_9 \sigma_1^6 + C_{10} \sigma_2^6
\quad \text{(15)}
\]

In the same manner as Eq.(9), when the following equation is assumed

\[
\phi^{(m)}(\sigma, \varepsilon) = \mu(\varepsilon) \phi^{(m)}_1(\sigma) + (1 - \mu(\varepsilon)) \phi^{(m)}_2(\sigma), \quad \text{(16)}
\]

it reduces to an interpolation for material parameters \( C_1, k, \ldots, N \), as follows

\[
C_k = \mu(\varepsilon) C_{k1} + (1 - \mu(\varepsilon)) C_{k0}, \quad \text{(17)}
\]

The present model of anisotropic hardening is validated by comparing the calculation results of flow stress directionality with the corresponding experimental data on AA6022-T43 aluminum sheet, as an example (see Figure 1).

![Figure 1. Flow stresses of AA6022-T43 aluminum sheet (experimental data are from Stoughton & Yoon [2]) and the predictions by the present model](image_url)

### 2.2 Kinematic hardening model with anisotropy evolution

Most kinematic hardening models assume the following form of the evolution equation of the back stress:

\[
\sigma = \frac{A}{Y}(\sigma - \alpha) - x \frac{\dot{\sigma}}{Y} \quad \text{(18)}
\]

\[
A = \frac{A}{Y}(\sigma - \alpha) - x \frac{\dot{\sigma}}{Y} \quad \text{. (18)}
\]
Here (\( \alpha \)) denotes the objective rate. In Eq. (8), the rate of workhardening in the kinematic hardening model is given by the following equation:

\[
H' = H'_{\alpha} = A - \frac{x}{\sigma_0} : x.
\]  

(19)

The Yoshida–Uemori model [7-9] was constructed within the framework of two-surface modeling, wherein the yield surface moves kinematically within a bounding surface, as schematically illustrated in Figure 1. To describe anisotropic hardening (i.e., expansion of the yield surface with shape change) and also kinematic hardening, the bounding surface \( F \) is expressed by the equation:

\[
F = \phi(\sigma - \beta, \varepsilon) - (B + R) = 0,
\]  

(20)

where \( \beta \) denotes the center of the bounding surface, and \( B \) and \( R \) are the initial size of the surface and its workhardening component, respectively. For details of evolution equations for \( \alpha \cdot \beta \) and \( R \), refer to papers [5-7]. To include the description of anisotropic hardening in the model, it is assumed that the shapes of both the yield and bounding surfaces vary simultaneously as a function of the effective plastic strain, as schematically illustrated in Figure 2.

![Figure 2. Schematic illustration of the changes of the yield surface and the bounding surface.](image)

To validate the model, cyclic stress-strain responses for a 780R AHSS sheet were examined. Cyclic strain tests and uniaxial tension tests were conducted using specimens cut from the sheet in three directions: \( 0^\circ \) (rolling direction), \( 45^\circ \) and \( 90^\circ \). The flow stresses and \( r \)-values of the three specimens were significantly different, depending on the sheet direction. The uniaxial tension test results showed that the stress directionality, measured with respect to normalized stresses, \( \sigma_{y_0} / \sigma_0 \) and \( \sigma_{y_0} / \sigma_0 \), varies with increasing plastic strain, whereas \( r \)-values do not change much throughout the deformation range. Figure 3 shows the anisotropic cyclic stress-strain calculated by the present model, where Yoshida’s six polynomial yield function [11] and the Yoshida-Uemori kinematic hardening model [5-7] were used.

3 Concluding remarks

The description of the Bauschinger effect and the evolution of anisotropy is made possible by incorporating the proposed anisotropic hardening model in a kinematic hardening model. This approach, which requires only one interpolation equation to describe the anisotropy evolution, offers a great advantage over other approaches in that it involves fewer material parameters \( A \) set of kinematic hardening parameters can be identified from experiments independent of anisotropic hardening parameters which define a yield function and its changing, and their values remain fixed throughout the plastic deformation.

![Figure 3. Anisotropic cyclic stress-strain responses on 780R-AHSS calculated by the present model (based on Y-U model).](image)

References