Comparison of two models for strain-path changes

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Abstract. Plastic anisotropic behavior after strain-path changes (SPCs) has been successfully predicted for steel and aluminium, respectively, by two recently proposed constitutive models. The two models are significantly different both with respect to the mathematical formulations and the physical meaning of the internal variables employed. The current work presents a comparison of the two models with respect to predicting the behavior of cast and homogenized aluminium AA1050. It is concluded that both models can capture nearly all the features of the material when subjected to reverse and orthogonal SPCs. However, there are various differences between the models with respect to prediction of permanent orthogonal softening and transient R-value evolution and the complexity of the parameter identification.

1 Introduction

The evolution of mechanical properties after SPCs has been predicted successfully by several constitutive models which are based on different physical assumptions. Therefore, a comparison of these models for one particular material is of great interest.

SPCs lead to transients in the stress-strain behavior, which in many cases will vanish after a certain strain, and then the monotonic loading behavior will prevail [1]. Examples are the Bauschinger effect and the cross hardening effect. In other cases, the stress-strain behavior may be permanently changed after the SPC. One such phenomenon is permanent softening [2].

Basically, there are two groups of models for changing the yield surface to capture the above properties: either isotropic hardening/softening combined with kinematic hardening or surface distortion.

In order to predict reverse softening and orthogonal hardening, Mánik et al. [3] proposed a constitutive model (here denoted KIH) based on the idea of isotropic hardening/softening combined with kinematic hardening. They used only second-order tensors to describe the evolution of the microstructure, and the interpretation of the model and its parameters is straightforward. The Bauschinger and cross hardening effects as well as the permanent softening after reverse and orthogonal loading can be captured.

The other group of models modifies the shape of the yield surface by distorting it. A general form of distortion of any type of isotropic or anisotropic yield surface was introduced by Barlat et al. [4], which is an alternative to kinematic hardening, and was denoted the Homogeneous Anisotropic Hardening (HAH) yield criterion. Lee et al. [5] used the model to characterize the springback during U-draw bend experiments and compared the results to those obtained with nonlinear kinematic hardening. The HAH model was then extended to cross-loading with latent hardening effects by use of dislocation-based modeling [6]. The extended HAH model was adopted by Ha et al. [7] to predict the Bauschinger effect and the transient hardening behavior of EDDQ and DP780 subjected to single SPC. An extension of the original HAH model was proposed to capture work-hardening stagnation and cross-hardening effects after orthogonal SPC [8].

The current work begins with a brief review of the formulation of the two constitutive models in Section 2. The parameter identification is outlined in Section 3. The stress-strain curves and R-values predicted by the two models are then compared in Section 4. Discussion and conclusions are presented in Section 5.

2 Model formulations

2.1 KIH model

The yield condition $f$ is written in the form

$$ f(S) = \varphi(S) - \sigma_f = 0 $$

$$ S = \sigma - X $$

$$ \sigma_f = R + S_e + S $$

where $S$ is overstress tensor, $\sigma$ is the stress tensor, $X$ is backstress tensor; $\sigma_f$ is the current yield stress, $R$ is the isotropic hardening variable, which contains the initial
yield stress, and the variables $S_s$ and $S_r$ represent the extra expansion and shrinkage of the yield surface after orthogonal and reverse SPC, respectively. The equivalent stress $\sigma = \varphi(S)$ is assumed to be a positive homogeneous function of order one. In this work, the non-quadratic anisotropic yield function Yld2000-2d is used.

### 2.2 HAH model

The yield function of the homogeneous anisotropic hardening model is formulated as

$$
\Phi(s) = \left( \sqrt{\psi(s)^2 + \psi'(s_s)^2} \right)^\gamma + f_1 \hat{h} : s - \hat{h} : s + f_2 \hat{h} : s + \hat{h} : s = \sigma
$$

where $\sigma$ is the equivalent stress. In Eq. (2), $\Phi(s)$ is a positive homogeneous yield function of order one in the deviatoric stress $s$, which is a combination of a stable component and a fluctuating component. Any isotropic or anisotropic yield function can be the stable component. The yield surface can be divided by the microstructure deviator into two half-planes. The stable component will control the half-plane to which the loading direction belongs, while the fluctuating component controls the other half-plane to control the Bauschinger effect. The stable component will also control the cross-hardening of the yield surface in the direction orthogonal to the microstructure deviator. Further, $\hat{h}$ is the microstructure deviator which memorizes the microstructure evolution history, while $q_1$, $f_1$, and $f_2$ are three coefficients. The yield function Yld2000-2d is used also here.

### 3 Parameter identification

The two models are fully implemented in a stand-alone program based on the MATLAB software, considering a single material point subjected to loading. A genetic algorithm is used to identify the parameters. The algorithm starts from a set of initial values for the parameters, and then these values are changed by a trial-and-error method guided by the physical meaning of the parameters. The identification processes of the two models are shown as Figure 1.

![Figure 1. Parameter identification processes of the two models](image)

### 4 Applications

Cast and homogenized commercially pure aluminium AA1050 was used to perform tensile tests pretrained by either compression or rolling to study the Bauschinger effect and cross-hardening effect, respectively. The material in this work is initially isotropic, and the coefficients of the Yld2000-2d yield function are all equal to unity.

Both models can capture the reverse softening, the stress plateau and the permanent softening effect after reverse SPC, as shown in Figure 2. However, there are minor differences between the stress-strain curves from the models and the experiment for a pre-strain of 0.02.

![Figure 2. Stress-strain curves for monotonic loading and compression-tension tests (reverse SPC) with 2% and 4.4% pre-strain](image)

As the HAH model does not predict the permanent softening after orthogonal SPC, the predicted stress-strain curves will always return to the monotonic loading curve. On the contrary, the KIH model has parameters dedicated to capture the permanent softening behavior. As shown in Figure 3, the KIH model can capture orthogonal hardening and subsequent permanent softening for pre-strain levels of 5% and 9.3%. For the 12.8 % pre-strain, the KIH model predicts permanent softening, while this is not observed experimentally.
Figure 3. Stress-strain curves for orthogonal SPC introduced by the sequence: rolling followed by uniaxial tension in transverse direction, after 5%, 9.3% and 12.8% of effective pre-strain. The monotonic loading curve in the transverse direction is also plotted for comparison.

Owing to the normality rule, the R-value, which is the width-to-thickness strain ratio in uniaxial tension, can be calculated from the gradient of the yield function. Thus, the R-value serves as an indicator on the evolution of the yield surface. The R-value predicted by the KIH model during the tensile tests after orthogonal strain-path changes with a pre-strain of 5% by rolling is decreasing in a similar way as the experimental data. In contrast, the R-value predicted by the HAH model deviates from the experimental data for the considered material, as shown in Figure 4.

![Figure 4](image-url)

Figure 4. Comparison of the modeled and measured R-values against true strain during the tensile tests after orthogonal strain-path changes with a pre-strain of 5% by rolling.

5 Discussion and conclusions

Both models can be applied to the AA1050 material. The predictions of the strain-stress curves obtained with the two models after SPC is acceptable. However, the two models are different in several aspects.

1. The evolution of the yield surface of the KIH model is shape invariant expansion/shrinkage combined with translation, while in the HAH model the yield surface is distorted.

2. The identification of the strain path is different, because different definitions of Schmitt factor are used. The microstructure deviator of the KIH model is formulated in the plastic strain rate space, whereas it is formulated in the deviatoric stress space in the HAH model.

3. The description of microstructure evolution is different. The microstructure deviator can change direction and magnitude in the KIH model, whereas it can only change direction in the HAH model.

4. The KIH model can capture the permanent softening behavior after orthogonal SPC, whereas the HAH model cannot.

5. The parameters are more easily identified for the HAH model because the parameters governing monotonic loading, reverse loading and orthogonal loading curves can be identified independently of each other. In contrast, the parameters describing reverse softening and orthogonal hardening in the KIH model influence the strain-stress behavior under monotonic loading.

References