A phase-field approach for ductile failure

Erfan Azinpour\textsuperscript{1,}\textsuperscript{a}, José César de Sá\textsuperscript{2} and Abel Santos\textsuperscript{3}

\textsuperscript{1}INEGI, Faculty of Engineering, University of Porto, Porto, Portugal
\textsuperscript{2}INEGI, Faculty of Engineering, University of Porto, Porto, Portugal

Abstract. Most of the stages of material behaviour can be described by means of continuous models but they fail to give an adequate answer when initiation and propagation of cracks occur. Continuous-discontinuous approaches can be adopted resorting to different techniques like element erosion, remeshing or X-FEM techniques. Here a new continuous model approach based on the phase-field theory is extended to ductile failure. The phase-field variable describes the damaged zone and its evolution is associated with standard damage models.

1 Introduction

Observation and prediction of fracture-induced failures has been a topic of intense research during the past decades in engineering applications. Notwithstanding the importance of experimental procedures to make observations during manufacturing processes to decrease the hazards of failure, utilization of computer-aided numerical approaches are of huge significance in order to reduce the excessive efforts and costs of such processes.

Recently, the extension of phase-field theory to failure design has gained a wide acceptance due to its better flexibility to confront the fracture processes compared to the continuous-discontinuous approaches like element erosion \cite{1} and eXtended Finite Element Method (XFEM) \cite{2} which suffer from pathological mesh dependency difficulties. Description of cracking as a phase-transition problem is considered as a key feature of this method which avoids the numerical tracking of sharp crack interfaces and the problem of discontinuous jumps of the displacement field. Moreover, it enables the approach to deal with complex crack pathology patterns such as multiple branching and merging crack situations in a more efficient manner.

Phase-field model of quasi-static brittle fracture was originally stemmed from the efforts of Frankfort and Marigo (1998) \cite{3} based on variational methods. This formulation was followed by the regularization of sharp crack paths into a transition zone via the phase-field variable performed by Boudin et al. (2000) \cite{4}. Later, Kuhn and Müller (2008) \cite{5} presented an additional model based on the Ginzburg-Landau type evolution equation which incorporated a mobility parameter. It is interesting to note that these authors were apparently the first ones who are responsible for using the phrase “phase-field model for fracture” in literature.

Since these models do not distinguish between the formation of crack surfaces under tensile and compressive loads, several numerical strategies proposed to do such a task. Regarding this, a model proposed by Amor et al (2009) \cite{6} to prevent the interpenetration of crack surfaces under compressive loads. Besides the analysis of quasi-static brittle fracture processes, dynamic phase-field fracture models have appeared in recent publications as addressed by Karma et al. \cite{7} and Mieh et al. \cite{8}.

Most recently and to avoid the pathological mesh dependency difficulties, the extension of the phase-field concept to the non-local, gradient enhanced damage models formerly proposed by Triantafyllidis and Aifantis (1986) \cite{9} has emerged as an interesting topic to explore. Following this, a novel computational strategy termed as the fast hybrid formulation was implemented by Ambati et al. (2015) \cite{10}. This model comprises both isotropic and anisotropic nature of previous models and reportedly reduced the computational cost of anisotropic phase-field fracture models up to 90\% as a consequence of keeping the linearity of momentum balance equations within the staggered time integration scheme.

2 Formulation

In the present contribution, the phase-field concept of ductile failure is extended to the phenomenological and micromechanical based local damage models. Thus, this section is intended to provide a concise review of the fundamentals of these concepts and also present the main idea behind the model which has been used in this work.

2.1 Phase-field concept

According to Griffith, the potential energy can be expressed as sum of the elastic energy density and the energy needed to create fracture surfaces namely the critical energy release rate ($G_c$) as follows

$$\psi_{pot}(\mathbf{u},\Gamma) = \int_\Omega \psi_e(\nabla \mathbf{u}) d\mathbf{x} + \int_\Gamma G_c d\mathbf{x}$$  \hspace{1cm} (1)

To tackle the problems associated with the numerical tracking of the propagating cracks, Bourdin et al. proposed the phase field parameter $\phi \in [0,1]$ where 0 and 1 represent the fully broken and virgin state of material respectively. To interpolate the sharp discontinuity surface denoted by $F$ within the transition zone, the total potential energy of Eq.1 is modified as follows

$$E_\phi(u, \phi) = \int_{\Omega} g(\phi) \psi_e(e(u))dx$$

$$+ \int_{\Omega} l_0 \left( \frac{1}{2\rho_0} (1-\phi)^2 + l_\phi (\nabla \phi)^2 \right)dx$$

(2)

where $l_0$ stands for the regularization parameter with a length dimension which controls the width of transition zone and $g(\phi) := \phi^2 + \eta$ is the general form of stress degradation function with a prescribed small dimensionless residual stiffness ($\eta$) to avoid numerical problems. Caution should be exercised to determine this parameter since adopting rather large values may result in the overestimated bulk energy on fractured zones.

According to variational principles, the strong form of the initial boundary value problem is expressed by the following equilibrium equation:

$$\text{div} \, \sigma(u, \phi) = 0$$

(3)

where $\sigma(u, \phi)$ is the second order Cauchy stress tensor as

$$\sigma(u, \phi) := g(\phi) \frac{\partial \psi(e)}{\partial e}$$

(4)

Correspondingly, the evolution of the phase-field variable can be obtained based on [5] as follows

$$\dot{\phi} = -M \frac{\partial E_\phi(e, \phi, \nabla \phi)}{\partial \phi}$$

$$= -M \left( \phi \epsilon : C : \epsilon - G_c \left( 2l_0 \partial \phi + \frac{1-\phi}{2l_0} \right) \right)$$

(5)

where $M$ describes the mobility of the process and with $M \rightarrow \infty$, the quasi-static crack propagation is recovered. This concept can be easily implemented using the standard finite element shape functions and by introducing the phase field as well as the displacement field as the active state variables.

### 2.2 Damage models

To date, a wide range of damage models have been presented to study the behaviour of structures in the presence of voids and micro-cavities. One of the phenomenological-based approaches which has inspired a lot of investigations is the model proposed by Jean Lemaitre (1983) [11]. This model uses the strain equivalence hypothesis to consider the internal deterioration of ductile material within the elasto-plastic framework. In regards with this concept, the elastic contribution to the free energy known as elastic-damage potential ($\psi^{ed}$) is expressed as

$$\rho \psi^{ed}(\epsilon^e, D) = \frac{1}{2} \epsilon^e : (1-D) C^e : \epsilon^e$$

(6)

where $D \in [0,1]$ is the damage variable. By considering the effective stress $\sigma_{eff} = C^e : \epsilon^e$, the Cauchy stress tensor is written as

$$\sigma = (1-D) C^e : \epsilon^e$$

(7)

and the thermodynamic force conjugate termed as the damage energy release rate can be obtained as follows

$$Y = \rho \frac{d \psi^{ed}}{d \phi} = \frac{1}{2} \epsilon^e : C^e : \epsilon^e$$

(8)

By introducing the integrity term, $\omega = (1-D)$, and disregarding the kinematic hardening variables, the simplified version of the Lemaitre damage model can be expressed by the following evolution relation

$$\dot{\gamma} = \frac{\lambda}{\omega} \left( \epsilon^e - \frac{f^*}{r^*} \right)$$

(9)

being $\dot{\gamma}$ the evolution of the plastic multiplier, whereas $r$ and $s$ are the damage evolution constants.

Another highly-notable approach which is instead based on the micro-mechanical aspects of material is the Gurson model (1977) [12] which has a wide applications in porous metals and ductile failure. This model is derived by assuming a spherical cavity embedded within the cubic rigid plastic matrix of material. The internal deterioration of the structure is defined as a relation between the volume of the void and the volume of the representative volume element (RVE):

$$\frac{V_m}{V_{RVE}} = (1-f)$$

(10)

being $f$ the void volume fraction or porosity. Later, this model was modified by Tvergaard and Needleman in which the evolution of porosity introduced and the coalescence of microvoids has been included in the original Gurson model. Thus, according to GTN model, the damage evolution is characterized by three successive mechanisms namely the evolution, growth and coalescence of microvoids and the effective porosity ($f^*$) is expressed by the following bilinear function

$$f^* = \begin{cases} \begin{align*}
\frac{f - f_c}{\frac{1}{2} - f_c} & , \quad f < f_c \\
\frac{f_c + \left( \frac{1}{2} - f_c \right)}{f_c} & , \quad f \geq f_c
\end{align*}
\end{cases}$$

(11)

where $f_c$ is the critical value of the volume void fraction and $f_f$ is the value of the volume void fraction at the final rupture of the material. The evolution of the porosity is characterized by the nucleation and growth mechanisms:

$$f = f^N + f^G$$

(12)

where

$$f^N = \frac{f_N}{S_N \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{\epsilon^p - \epsilon_{N}}{S_N} \right)^2 \right) \hat{\epsilon}^p$$

(13)

and

$$f^G = \left( 1 - f \right) \hat{\epsilon}^p$$

(14)

In Eq. (13), $f_N$ represents the volume void fraction of all particles with potential for microvoid nucleation, $\epsilon_N$ and $S_N$ are the mean strain for void nucleation and its standard deviation, $\hat{\epsilon}^p$ is the equivalent plastic strain and $\hat{\epsilon}^p$ is the accumulated plastic strain. The yield function of the GTN damage model is given by the following relation

$$\phi(\sigma, k, f) =$$
where \( J_2(s) \) is the second invariant of the deviatoric stress tensor and \( q_1, q_2 \) and \( q_3 \) are the optimizing variables introduced by Tvergaard to bring the model predictions in a closer agreement with the real conditions. The difficulties to choose appropriate values for these parameters is known as a drawback of this approach.

### 2.3 Phase-field local damage models

As stressed out before, this study concerns the introduction of the phase-field approach into a local damage framework. To perform this task, an attempt is made to bring the phase-field parameter within the constitutive equations of the damage models based on Lemaitre and GTN methods reviewed in the previous section.

Regarding the Lemaitre damage model, the damage variable highly resembles the phase-field parameter and the following relation can be obtained

\[
D = 1 - \varphi
\]  

whereas the phase-field approach arises at the crack interfaces (\( \varphi \rightarrow 1 \)), the internal damage parameter decays to one (\( D \rightarrow 1 \)) to fulfill the equation. By modifying Eq. (9) according to Eq. (16), the evolution of the phase-field is given by

\[
\dot{\varphi} = \frac{\gamma}{\varphi} \left( \frac{\gamma}{\varphi} \right) ^s
\]  

The second part of the work involved with the introduction of the phase-field approach into the GTN damage model. Since porosity is the key parameter that triggers the softening regime of structure in such model, the idea is to associate the phase-field parameter with this variable according to the following expression:

\[
\frac{f}{f_c} = 1 - \varphi
\]  

in which at the fully-broken material state, the effective porosity equals the critical value of the volume void fraction. Thus, blending the phase-field approach with the GTN constitutive equations leads to the following evolution equation:

\[
\dot{\varphi} = -\frac{1}{f_c} \left( \varphi^N + \varphi^G \right)
\]  

where

\[
\varphi^N = \frac{f_N}{s_N V_0} \exp \left( -\frac{1}{2} \left( \varphi^P + \varphi^G \right) \right)
\]

and

\[
\varphi^G = \left[ 1 - f_c (1 - \varphi) \right] \dot{\varepsilon}_V^G
\]

In Eq. (19), \( \varphi^N \) and \( \varphi^G \) stand for the evolution of the phase-field due to nucleation and growth of the voids in the matrix of the material.

Throughout this study, the finite element framework is used as the main numerical method to implement these concepts. In order to solve the nonlinear equilibrium and evolution equations for the desired state variables iteratively, a staggered time integration scheme is utilized due to its higher robustness and effectiveness compared to the monolithic time integration approaches. At the final stage and in order to evaluate the performance and convergence rates of the proposed concepts, several benchmark examples are analyzed.

### Acknowledgement

Authors gratefully acknowledge the funding of Project NORTE-01-0145-FEDER-000022 - SciTech - Science and Technology for Competitive and Sustainable Industries, cofinanced by Programa Operacional Regional do Norte (NORTE2020), through Fundo Europeu de Desenvolvimento Regional (FEDER).

### References


